Counting graphs without a fixed subgraph

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A graph is called H-free if it contains no copy of H. Denote by $f_n(H)$ the number of (labeled) H-free graphs on n vertices. Since every subgraph of an H-free graph is also H-free, it immediately follows that $f_n(H) \geq 2^{\mathrm{ex}(n,H)}$. Erdős conjectured that, provided H contains a cycle, this trivial lower bound is in fact tight, i.e.

$$f_n(H) = 2^{(1+o(1))\,\mathrm{ex}(n,H)}.$$

The conjecture was resolved in the affirmative for graphs with chromatic number at least 3 by Erdős, Frankl and Rödl (1986), but the case when H is bipartite remains wide open. We will give an overview of the results in case $\chi(H) = 2$, talk about a few related problems and present our recent contributions to the study of H-free graphs. This is joint work with József Balogh.