

Avoiding small subgraphs in the Achlioptas process

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Abstract

The standard paradigm for online power of two choices problems in random graphs is the Achlioptas process. Here we consider the following natural generalization: Starting with G_0 as the empty graph on n vertices, in every step a set of r edges is drawn uniformly at random from all edges that have not been drawn in previous steps. From these, one edge has to be selected, and the remaining $r - 1$ edges are discarded. Thus after N steps, we have seen rN edges, and selected exactly N out of these to create a graph G_N . In a recent paper by Krivelevich, Loh, and Sudakov (Rand. Struct. Alg. 2009), the problem of avoiding a copy of some fixed graph F in G_N for as long as possible is considered, and a threshold result is derived for some special cases. Moreover, the authors conjecture a general threshold formula for arbitrary graphs F . We disprove this conjecture and give the complete solution of the problem by deriving explicit threshold functions $N_0(F, r, n)$ for arbitrary graphs F and any fixed integer r .