Peter Heinig: Half of the minimum degree suffices for bipartite graphs to contain spanning subgraphs of low bandwidth

Bollobás and Komlós conjectured that for all $\gamma > 0$ and all positive integers r and Δ, there exists $\beta > 0$ and a positive integer n_0 such that every graph G with n vertices and minimum degree at least $(1 - \frac{1}{r} + \gamma)n$ contains *every* graph H on n vertices that is r-colourable, has bandwidth at most βn and maximum degree at most Δ as a (spanning) subgraph. This has been proven by Böttcher, Schacht, and Taraz [Math. Ann 343(1), 175–205, 2009].

This theorem has a rather simple hypothesis for the host graph whereas the assumptions about the spanning subgraph are more complicated. We are interested in how far additional structural hypotheses about the host graph can lead to strengthenings of the above result. In this spirit, we recently proved that the minimum degree threshold can be cut in half if the host graph is a balanced bipartite graph. It turned out that it is not possible to simply imitate the proof of the Bollobás-Komlós conjecture with r specialized to 2. Ironically, the additional assumption about the host graph made it necessary to develop a new idea about how to process the *sub*graph during the embedding procedure, while the host graph is handled essentially in the same way as in the proof of the Bollobás-Komlós conjecture. (Joint work with Julia Böttcher and Anusch Taraz.)