Strange behavior of Avoider and Enforcer

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An Avoider-Enforcer game is played by two players, called Avoider and Enforcer, on a hypergraph $\mathcal{F} \subseteq 2^X$. The players claim previously unoccupied elements of the board X in turns. Enforcer *wins* if Avoider claims all vertices of some element of $\mathcal{F}$, otherwise Avoider wins. In a more general version of the game a *bias* b is introduced to level up the players' chances of winning; Avoider claims one element of the board in each of his moves, while Enforcer responds by claiming b elements. This traditional set of rules for Avoider-Enforcer games is known to have a shortcoming: it is not bias monotone.

We relax the traditional rules in a rather natural way to obtain bias monotonicity. We analyze this new set of rules and compare it with the traditional ones to conclude some surprising results. In particular, we show that under the new rules the threshold bias for both the connectivity and Hamiltonicity games, played on the edge set of the complete graph K_n, is asymptotically equal to $n/\log n$. This coincides with the asymptotic threshold bias of the same game played by two "random" players.

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