Triangle-free subgraphs at the triangle-free process

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Abstract

We consider the triangle-free process: given an integer n, start by taking a uniformly random ordering of the edges of the complete n-vertex graph K_n. Then, traverse the ordered edges and add each traversed edge to an (initially empty) evolving graph - unless its addition creates a triangle. We study the evolving graph at around the time where $\Theta(n^{3/2+\varepsilon})$ edges have been traversed for any sufficiently small fixed ε. At that time and for any fixed triangle-free graph F, we give an asymptotically tight estimation of the expected number of copies of F in the evolving graph. For F that is balanced and has density smaller than 2 (e.g., for F that is a cycle of length at least 4), our argument also gives a tight concentration result for the number of copies of F in the evolving graph. Our analysis combines Spencer's original branching process approach for analysing the triangle-free process and the semi-random method.

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