

# The tripartite Ramsey number for trees

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RSA – Poznań, August 3-7, 2009

## Abstract

It is known that the Ramsey number  $R(\mathcal{T}_{t+1}, \mathcal{T}_{t+1})$  of the class of trees of order  $t+1$  is at most  $2t$  (for large  $t$ ). Schelp conjectured that if, instead of the complete graph, we 2-colour the balanced complete tripartite graph on  $2t$  vertices, then in one of the colours we find a copy of any bounded-degree tree of order  $t - o(t)$ . We give a solution of this conjecture for a sublinear bound on the maximal degree of the trees.

**Theorem 1** (J. Böttcher, J. Hladký, D. P.). *For all  $\varepsilon > 0$  there are  $\alpha > 0$  and  $n_0 \in \mathbb{N}$  such that for all  $n \geq n_0$  the following holds. For any two-colouring of the edges of  $K_{n,n,n}$  one colour contains copies of all trees  $T$  of order  $t \leq (3 - \varepsilon)n/2$  and with maximum degree  $\Delta(T) \leq n^\alpha$ .*

The proof uses regularity lemma and the notion of connected matching in the reduced graph. Connected matching has proved to be useful for embedding balanced trees of bounded degree. We generalised this notion to *connected fork-system* to handle the embedding of bounded-degree unbalanced trees.