

# Removal Lemma for systems of linear equations

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## Abstract

We study algebraic analogues of the graph Removal Lemma. In 2005, Green conjectured the following analogue of it for systems of equations over integers:

For every  $k \times m$  integral matrix  $A$  with rank  $k$  and every  $\varepsilon > 0$ , there exists  $\delta > 0$  such that the following holds for every  $N$  and every  $S \subseteq \{1, \dots, N\}$ : if the number of solutions of  $Ax = 0$  with  $x \in S^m$  is at most  $\delta N^{m-k}$ , then it is possible to destroy all solutions  $x \in S^m$  of  $Ax = 0$  by removing at most  $\varepsilon N$  elements from the set  $S$ .

We prove this conjecture by establishing its variant for not necessarily homogenous systems of equations over finite fields. The core of our proof is a reduction of the statement to the colored version of hypergraph Removal Lemma for  $(k+1)$ -uniform hypergraphs. Independently of us, Shapira obtained the same result using a reduction to the colored version of hypergraph Removal Lemma for  $O(m^2)$ -uniform hypergraphs.