

Removal Lemma for systems of linear equations

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Abstract

We study algebraic analogues of the graph Removal Lemma. In 2005, Green conjectured the following analogue of it for systems of equations over integers:

For every $k \times m$ integral matrix A with rank k and every $\varepsilon > 0$, there exists $\delta > 0$ such that the following holds for every N and every $S \subseteq \{1, \dots, N\}$: if the number of solutions of $Ax = 0$ with $x \in S^m$ is at most δN^{m-k} , then it is possible to destroy all solutions $x \in S^m$ of $Ax = 0$ by removing at most εN elements from the set S .

We prove this conjecture by establishing its variant for not necessarily homogenous systems of equations over finite fields. The core of our proof is a reduction of the statement to the colored version of hypergraph Removal Lemma for $(k + 1)$ -uniform hypergraphs. Independently of us, Shapira obtained the same result using a reduction to the colored version of hypergraph Removal Lemma for $O(m^2)$ -uniform hypergraphs.