

Playing to retain the advantage

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Abstract

Let P be a monotone decreasing graph property, let $G = (V, E)$ be a graph, and let q be a positive integer. In this paper, we study the $(1 : q)$ Maker-Breaker game, played on the edges of G , in which Maker's goal is to build a graph that does not satisfy the property P . It is clear that in order for Maker to have a chance of winning, G must not satisfy P . We prove that if G is far from satisfying P , that is, if one has to delete many edges from G in order to obtain a graph that satisfies P , then Maker has a winning strategy for this game. We also consider a different notion of being far from satisfying some property, which is motivated by a problem of Duffus, Luczak and Rödl.