

# Playing to retain the advantage

Dan Hefetz

## Abstract

Let  $P$  be a monotone decreasing graph property, let  $G = (V, E)$  be a graph, and let  $q$  be a positive integer. In this paper, we study the  $(1 : q)$  Maker-Breaker game, played on the edges of  $G$ , in which Maker's goal is to build a graph that does not satisfy the property  $P$ . It is clear that in order for Maker to have a chance of winning,  $G$  must not satisfy  $P$ . We prove that if  $G$  is far from satisfying  $P$ , that is, if one has to delete many edges from  $G$  in order to obtain a graph that satisfies  $P$ , then Maker has a winning strategy for this game. We also consider a different notion of being far from satisfying some property, which is motivated by a problem of Duffus, Luczak and Rödl.