Average Case Analysis of a Class of NP-Complete Problems:

Maximum Independent Set and Exhaustive Search Algorithms.

Cyril Banderier, Vlady Ravelomanana (Univ. Paris Nord)

& Hsien-Kuei Hwang, Vytas Zacharovas (Academica Sinica)

In this talk, we deal with rigorous average case analysis of a large class of NP complete problems, relying on some mathematical tools from complex analysis and probability theory. We consider here one of the historical prototype of such problems: Maximum Independent Set (MIS), but our results apply more generally to a whole family of algorithms for constraint satisfaction problems (including reduction algorithms: Tarjan-Chv\`atal like algorithms, or classical exhaustive algorithms which encompass 3-coloring, Max Cut, Max 2-SAT, ...).

Our approach gives a precise picture of the ``complexity landscape'' of these algorithms, depending on the average degree (or the ratio vertices/edges) of the graph, and gives access to the location of the ``hard regions'' where people could then sample their inputs if they want to make some benchmarks (may it be for the worst or average case). Under the $\Gamma(n,m)$ and $G(n,p)$ distribution for graphs, we give some fascinating phase transitions between exponential (A^n), superpolynomial ($n^{\ln n}$), and polynomial (n^d) average complexities.

The challenging associated mathematical aspects force us to introduce new analyses (for a large class of Mahlerian-like recurrences), which will clearly be of interest for many other NP hard problems (typically, optimization problems on graphs).