

# Novel Characteristics of Split Trees by use of Renewal Theory

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## Abstract

We introduce the use of renewal theory in studies of split trees.  
*(These are a large class of random trees of logarithmic height first described by Devroye. A split tree of cardinality  $N$  is obtained by distributing  $N$  “balls”, which often represent some “key numbers”, generating a subset of vertices in an infinite tree.)*

Using renewal theory we give a relation between the deterministic number of balls  $N$  and the random number of vertices  $n$ .

We present several results for the depths of the balls and the vertices in the tree, e.g.

- *There is a central limit law for the depths of the balls, so that most vertices are close to a depth  $c \ln N$ , for some constant  $c$ .* We sharpen this result, by finding a small upper bound for the expected number of vertices that are outside a region close to  $c \ln N$ .

- We find the first asymptotic of the variances for the depths of the balls in the tree.