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Graph eating game

Alice and Bob share a connected graph. Its vertices are weighted with non-negative values summing up to one. The players eat the vertices alternately one by one (starting with Alice) until no vertex is left.

The rule they have to obey is that after each move the vertices eaten so far form a connected subgraph of the original graph. Both players want to maximize their final gain, i.e., the total weight of the vertices they have eaten. This game for a cycle is known as the pizza eating problem. Recently, Knauer, Micek and Ueckerdt proved that Alice can eat $4/9$ of any cycle (pizza), which is best possible and settles the conjecture of Peter Winkler.

In the general game, Alice cannot guarantee herself any positive constant gain on all connected graphs. Curiously, the parity of the number of vertices makes a difference. Examples of graphs with small Alice's gain having an odd number of vertices need a very rich structure, contrary to strikingly simple examples with an even number of vertices. In particular, there are trees with an even number of vertices which are very bad for Alice, while she can guarantee herself a positive constant gain on all odd trees.

We wish to introduce the audience to this and similar games on graphs. Our techniques are quite general and seem to be applicable to other combinatorial games as well.