Quasirandom rumour spreading on the complete graph

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Abstract

Randomized rumour spreading is a protocol for disseminating information in a graph. Initially there is one vertex which holds a piece of information and during each round every one of the informed vertices chooses one of its neighbours uniformly at random and independently and informs it. Doerr, Friedrich and Sauerwald (2008) introduced a quasirandom version of this method, where each vertex has a cyclic list of its neighbours. Once a vertex has been informed, it chooses uniformly at random only one neighbour. In the following round, it informs this neighbour and at each subsequent round it picks the next neighbour from its list and informs it. The randomized model was initially studied by Frieze and Grimmett (1985) on the complete graph with n vertices, where they proved that for every $\varepsilon > 0$ all nodes are informed within $(1 \pm \varepsilon)(\log_2 n + \ln n)$ stages with probability $1 - o(1)$. Pittel (1987) improved on this, showing that in fact for any ω such that $\omega(n) \to \infty$ as $n \to \infty$ the randomized broadcasting protocol informs all vertices within $\log_2 n + \ln n \pm \omega(n)$ stages with probability $1 - o(1)$. Recently, Angelopoulos, Doerr, Huber and Panagiotou proved the analogue of the result which Frieze and Grimmett proved for the randomized model on the complete graph.

We give a precise analysis of the evolution of the quasirandom protocol on the complete graph with n vertices and show that it evolves essentially in the same way as the randomized protocol. In particular, we prove an (almost) analogue of the bound that Pittel gave for the randomized model. That is, if $S(n)$ denotes the number of rounds that are needed until all vertices are informed, we show that for any ω such that $\omega(n) \to \infty$ as $n \to \infty$ one has

$$\log_2 n + \ln n - 4 \ln \ln n \leq S(n) \leq \log_2 n + \ln n + \omega(n),$$

with probability $1 - o(1)$.
This is joint work with Nikolaos Fountoulakis.