

On K_s -free subgraphs in K_{s+k} -free graphs

Andrzej Dudek, Carnegie Mellon University

Extending the problem of determining Ramsey numbers Erdős and Rogers introduced the following function. For given integers $2 \leq s < t$ let $f_{s,t}(n) = \min \{ \max \{ |S| : S \subseteq V(H) \text{ and } H[S] \text{ contains no } K_s \} \}$, where the minimum is taken over all K_t -free graphs H of order n . This function attracted a considerable amount of attention despite which the gap between the lower and upper bounds remains fairly wide. For example, when $t = s + 1$, the best bounds have been of the form $\Omega(n^{\frac{1}{2}+o(1)}) \leq f_{s,s+1}(n) \leq O(n^{1-\epsilon(s)})$, where $\epsilon(s)$ tends to zero as s tends to infinity. This raised the following question asked by Krivelevich and later by Sudakov. Is it true that for every $0 < \delta < 1$ and s sufficiently large $f_{s,s+1}(n)$ is greater than $n^{1-\delta}$? In this talk, we show that this is not the case. We improve the upper bound by showing that $f_{s,s+1}(n) \leq O(n^{\frac{2}{3}})$ for every $s \geq 2$. Moreover, we show that for every $\varepsilon > 0$ and sufficiently large integers $1 \ll k \ll s$, $\Omega(n^{\frac{1}{2}-\varepsilon}) \leq f_{s,s+k}(n) \leq O(n^{\frac{1}{2}+\varepsilon})$.

This is joint work with Vojta Rödl.